

Relative Orbital Integrals

We have worked in the "spectral-side". Now we work on the "geometric side". (Using algebraic geometry?)

Algebraic groups. Let G be an algebraic group over a field F .

• (Points P.O.V) Given a R a F -algebra; then

$$R \mapsto G(R) = \text{Hom}_F(F[G], R)$$

Remark "Yoneda lemma" says that the R -points for every R characterize G .

e.g. i) $X := \text{Spec} (\mathbb{R}[x, y] / (x^2 + y^2 + 1))$.

- $X(\mathbb{C}) \cong \{ (z_1, z_2) / z_1^2 + z_2^2 + 1 = 0 \}$
- $X(\mathbb{R}) = \emptyset$

ii) $Y = \text{Spec}(\mathbb{R}[x, y] / (xy - 1))$

- $Y(\mathbb{C}) \cong \{(z_1, z_2) : z_1 z_2 = 1\} \cong \mathbb{C}^\times$

- $Y(\mathbb{R}) \cong \mathbb{R}^\times$

Visual closed points

$X =$ 

$Y =$ 

Suppose that R is a locally compact ring, then we can endow G .

- $A_F^N(R) = R^N \rightarrow$ Product topology.

- $G \xrightarrow{\text{closed}} A_F^N \rightarrow G(A_F) \subset R^N$ induced topology.