

Exm 1. \square quotients: $X \rightarrow Y$ surjective $\not\Rightarrow X(F) \rightarrow Y(F)$
 surjective
 But if $F = \bar{F}$, it is true.

In particular $H_1^H(F) \neq H_1(F) \setminus H(F)$

Prop let I be an affine smooth alg. subgroup of H .

Then

$$(I \setminus H)(F) = \{ I(F^{\text{sep}}) h \in I(F^{\text{sep}}) \setminus H(F^{\text{sep}}) :$$

$$h \cdot \zeta(h^{-1}) \in I(F^{\text{sep}}) \forall \zeta \in \text{Gal}_F \}$$

(Ideal) Smooth: $X = \text{Spec} \left(\mathbb{F}[T_1, \dots, T_n] \Big/_{(f_1, \dots, f_m)} \right)$ is smooth if $\det \left(\frac{\partial f_i}{\partial T_j} \right)_{\substack{i, j \in \{1, \dots, m\}}}^{-1}$ is invertible. $n \geq m$.

Prop If X is smooth then $X(\mathbb{F}^{\text{sep}})$ is dense in X .

Rmk Compare to: $X(\mathbb{F}) \subset X$ dense. But \mathbb{F}^{sep} -points is better since

$$X(\mathbb{F}) = X(\mathbb{F}^{\text{sep}})^{\text{Gal}(\mathbb{F}^{\text{sep}}/\mathbb{F})} \quad (\text{Galois gluing...})$$

• Non-example: $X = \text{Spec} \left(\mathbb{F}_p[T] \Big/_{Y^p - T} \right)$

$$\frac{\partial(Y^p - T)}{\partial Y} = p Y^{p-1} = 0$$

$$X(\mathbb{F}^{\text{sep}}) = \text{Hom} \left(\mathbb{F}_p[T] \Big/_{Y^p - T}, \mathbb{F}^{\text{sep}} \right)$$

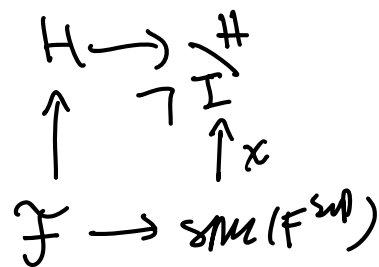
$$= \{ a \in \mathbb{F}^{\text{sep}} : a^p - T = 0 \} = \emptyset$$

↳ Not sep min. Polynomials.

• We seek to give a description of $\frac{H}{I}(F^{sep})$. We indeed prove that $\overline{\frac{H}{I}(F^{sep})} = \overline{I(F^{sep}) \setminus H(F^{sep})}$:

$\frac{H}{I}(F^{sep}) \rightarrow \overline{\frac{H}{I}(F^{sep})}$ is surjective.

Fix $x \in \overline{\frac{H}{I}(F^{sep})}$.



Is $\mathbb{F} \neq \emptyset$? • $\mathbb{F}(\overline{\mathbb{F}}) \neq \emptyset$

- $\mathbb{F} \supset I \cdot h$; $h \in H(\overline{\mathbb{F}})$.
- But $I \cdot h$ is smooth

Prop $\Rightarrow I \cdot h|_{\mathbb{F}} \neq \emptyset$
 \Downarrow
 $\mathbb{F}(h) \neq \emptyset$.

• By Galois theory we get the result.

Rmh let I_h be the stabilizer of $I(F^{\text{sep}})_h$ under the action of H . Then

$$(I/H)(F) = \coprod_{\substack{h \\ H(F)\text{-orbits in } (I/H)(F)}} \text{Im}(I_h(F) \backslash H(F) \rightarrow (I/H)(F)),$$

Note that I_h is a (inner) form of I , i.e.,

$$I_{F^{\text{sep}}} \cong I_{h, F^{\text{sep}}}$$

(Galois cohomology)