Rmk. • ② Quotients: $X \to Y$ surjective $\not\Rightarrow$ $X(F) \to Y(F)$ surjective

But if $F = \overline{F}$, it is true.

In particular $H_\lambda \backslash H(F) \neq H_\lambda(F) \backslash H(F)$

Prop Let I be an affine smooth alg. subgroup of H.
Then

$$(I \backslash H)(F) = \{ I(F^{sep}) h \in I(F^{sep}) \backslash H(F^{sep}) : h \cdot \xi(h^{-1}) \in I(F^{sep}) \ \forall \xi \in Gal_F \}$$

(Idea) Smooth : $X = \mathrm{Spec}\left(F[T_1, \ldots, T_n] / (f_1, \ldots, f_m)\right)$ is smooth ($n \geq m$)

if $\det\left(\frac{\partial f_i}{\partial T_j}\right)_{1 \leq i \leq m}$ invertible.

Prop If X is smooth then $X(F^{sep})$ is dense in X.

Rmk · Compare to: $X(F) \subset X$ dense. But F^{sep}-points is better since

$X(F) = X(F^{sep})^{\mathrm{Gal}(F^{sep}/F)}$ (galois gluing ...)

· Non-example : $X = \mathrm{Spec}\left(\underbrace{\mathbb{F}_p(T)}_{F}[Y] / Y^p - T\right)$

$$\cdot \quad \frac{\partial(Y^p - T)}{\partial Y} = pY^{p-1} = 0$$

$$\cdot \quad X(F^{sep}) = \mathrm{Hom}\left(\mathbb{F}_p(T)[Y] / Y^p - T, F^{sep}\right)$$

$$= \{\alpha \in F^{sep} : \alpha^p - T = 0\} = \phi$$

↳ Not sep min. polynomial.

• We seek to give a description of $I\backslash H(F^{sep})$. We indeed prove that

$$\boxed{I\backslash H(F^{sep}) = I(F^{sep})\backslash H(F^{sep})}:$$

$$\Updownarrow$$

$$\boxed{H(F^{sep}) \longrightarrow I\backslash H(F^{sep}) \text{ is surjective.}}$$

Fix $x \in I\backslash H(F^{sep})$.

$$\begin{tikzcd} H \arrow[r] & I\backslash H \\ \mathcal{F} \arrow[u] \arrow[r] \arrow[ur, phantom, "\urcorner"] & \operatorname{spec}(F^{sep}) \arrow[u, "x"'] \end{tikzcd}$$

is $\mathcal{F} \neq \emptyset$?

· $\mathcal{F}(\bar{F}) \neq \{0\}$

· $\mathcal{F} \supset I\cdot h$; $h \in H(\bar{F})$.

· But $I\cdot h$ is smooth

Prop $\Rightarrow$ $I\cdot h(k_s) \neq \emptyset$

$\Downarrow$

$\mathcal{F}(k_s) \neq \emptyset$.

· By Galois gluing we get the result.

Rmk Let I_h be the stabilizer of $I(F^{sep})h$ under the action of H. Then

$$(I\backslash H)(F) = \coprod_{\substack{h \\ H(F)\text{-orbits in } (I\backslash H)(F)}} \operatorname{Im}\left(I_h(F)\backslash H(F) \longrightarrow (I\backslash H)(F)\right),$$

Note that I_h is a (inner) form of I, i.e.,

$$I_{F^{sep}} \simeq I_{h,F^{sep}}$$

(Galois cohomology)