Recall

- (quotient) $(I\backslash H)(F) = \coprod_{h} \mathrm{Im}\left(I_h(F)\backslash H(F) \longrightarrow (I\backslash H)(F)\right)$,

  $H(F)$-orbits in $(I\backslash H)(F)$

  Note that I_h is a (inner) form of I, i.e.,

  $$I_{F^{sep}} \simeq I_{h, F^{sep}}$$

- (Topology) If F is locally compact field then
  - $\mathbb{A}^n_F \rightsquigarrow \mathbb{A}^n_F(\mathbb{R}) = \mathbb{R}^n$ (Product Topology)
  - $G \overset{\text{closed}}{\hookrightarrow} \mathbb{A}^n_F \rightsquigarrow G(\mathbb{R}) \hookrightarrow \mathbb{R}^n$ (Topology)?

H smooth affine algebraic group and I a smooth sbg.

Thm $H(F) \longrightarrow \left(I\backslash H\right)(F)$ has a closed image
$\quad I(F)$

Idea: One can use Remark above to prove reduce to prove that $H \to {}_I\backslash H$ has an open image. But $H \to {}_I\backslash H$ is smooth,

Thus $H(F) \to {}_I\backslash H(F)$ has open image.

(Relative Orbitals Integrals) Let H be a smooth alg. group defined over F acting on smooth affine scheme X.

Fix $\chi: H(F) \to \mathbb{C}^\times$ quasi-character

- $\gamma \in X(F)$ is **χ-relevant** if χ is trivial on $H_\gamma(F)$.
- $C_c^\infty(X)$ locally compact supported (smooth) locally constant on $X(F)$ (F non-archimedean).
- $\gamma \in X(F)$ is **relatively unimodular** if $\delta_H|_{H_\gamma} = \delta_{H_\gamma}$. ↪ Modulus character...

form $\frac{d_r h}{d_r h_\gamma}$.

$$RO_\gamma(f) = RO_\gamma^\chi(\gamma) = \int_{H_\gamma(F)\backslash H(F)} f(\gamma h)\chi(h)\, \frac{d_r h}{d_r h_\gamma}.$$

Thm If $\gamma \in X(F)$ is relevant, relatively unimodular and relatively semisimple and H_γ is smooth then $RO_\gamma(f)$ is absolutely convergent

We say γ is relatively semi-simple if the reduced image $O(\gamma)$ of $H \xrightarrow{\alpha(-\gamma)} X$ is closed.

Proof (Idea) Enough to prove that the pull back

$$C(X(F)) \to C(H_\gamma(F)\backslash H(F))$$

through $H_\gamma(F)\backslash H(F) \to X(F)$ is proper.

- relatively semi-simple $\overset{\text{reduce}}{\Longrightarrow}$ $X = H_\gamma \backslash H$.
- Thm $\Rightarrow$ $H_\gamma(F)\backslash H(F) \xrightarrow{\text{image}} (H_\gamma\backslash H)(F)$ is closed.