Thm Suppose that $\chi|_A = 1$. The expression

$$\int_{AH(F)\backslash H(\mathbb{A}_F)} \sum_\gamma |f(\gamma\cdot h)\chi(h)|\, dh.$$

- $X = H$-space
- $H = H_r \times H_u$
- C_x = stabilizer of x under H_r.
  - $\hookrightarrow A = A_{C_x}$

is convergent. Here the sum is over $\gamma \in X(F)$ that are Galois cohomologically finite, relative unimodular and relative elliptic, and such that the H_r-orbit is closed.

gcf: H_γ is smooth and $|H^1(F_v, H_\gamma/H_\gamma^0)| < \infty$ $\forall v$.

- F is a number field then is gcf (char $F=0$ $\Rightarrow$ affine alg. groups are smooth.)

r.e. H_γ smooth and $H^0_{\gamma,r} = H_\gamma$:
γ relatively elliptic if and only if $A_{H_\gamma} \subset A$.

Note also that if $H^0_{\gamma,u} = H^0_\gamma$ then

$H_{\gamma,u}(\mathbb{A}_F)/H_{\gamma,u}(F)$ compact.

- H_γ/H_γ^0 = finite group (scheme)

$H^1 \hookrightarrow \mathrm{Hom}(\mathrm{Gal}_{F_v}, \Gamma)/$conj.

suppose Γ is ab. $\quad \mathrm{Hom}_{cts}(K^\times, \Gamma)$

Proof: Idea We assume that $H = H^0$ (H/H^0 is finite).

One can show that (the adjectives are transported via the H-action.)

$$\int_{A_H(F)\backslash H(\mathbb{A}_F)} \sum_{\gamma} |f(\gamma \cdot h)\, \chi(h)|\, dh = \sum_{\mathcal{O}} \underbrace{\int_{A_H(F)\backslash H(\mathbb{A}_F)} \sum_{\gamma \in \mathcal{O}(F)} |f(\gamma\cdot h)\chi(h)|\, dh}_{*}.$$

where the sum is over closed H-orbits in X, s.t. $\forall \gamma \in \mathcal{O}(F)$ has the closed orbit and get relatively unimodular and relatively elliptic.

$\mathcal{O} \simeq H/H_\gamma$ $\qquad (*) = \sum_{\substack{\gamma \\ H(F)\text{-orbit} \\ \text{in } \mathcal{O}(F)}} \int_{A_{H_\gamma}(F)\backslash H(\mathbb{A}_F)} |f(\gamma\cdot h)\chi(h)|\, dh$

$$= \sum_{\gamma} \underbrace{\tau(H_\gamma)}_{<\infty} RO_\gamma^{\chi}(|f|)$$